International Distributions of the Age at Death and Mortality Convergence

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Abstract

Near-linear increases in life expectancy at birth in industrialized countries during the postwar period have been interpreted by some as indicating convergence to a single, universal mortality schedule. Although there has been much convergence in the mean age at death conditional on surviving to any age, convergence in the variance of the age at death has followed a different pattern. Universal reductions in infant and child mortality have raised life expectancies and lowered unconditional variances of the age at death, but there are differential trends in variances in the age at death conditional on early survival among advanced economies. Vastly different patterns of later-life variance in the age at death during the postwar period are suggestive of differential individual-level heterogeneity in health outcomes that may be connected to differential mortality decline.

1 Introduction

Mortality rates have dropped dramatically in every industrialized country since World War II. In order to assess possible future patterns of mortality

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decline, researchers have sought to characterize and explain these postwar developments. Although the entire distribution of deaths is obviously important to the individuals who experience it, much previous work on aggregate mortality has focused exclusively on the mean age at death, or period life expectancy at birth.

White (2002) finds linear increases in period life expectancy at birth for both sexes combined among high-income countries in the postwar period. Increases average roughly 1 extra year of life for every 5 years of time. White also reports evidence of convergence in period life expectancy among high-income countries and therefore posits that they are increasingly adhering to a single mortality schedule. In a related study, Oeppen and Vaupel (2002) find linear increases in "best practices" or record female life expectancy among industrializing countries in the 160 years since 1840. Annual rates of increase over this longer historical period averaged almost 1 year of life every 4 years.

The mean age at death is the most widely referenced moment of the mortality distribution for good reason: by definition it is of first order importance. But there is considerable variation around the mean age at death in human populations, especially during high-mortality historical periods or in less developed countries where mortality rates are high. Trends in variability are likewise as important as trends in mean ages at death. Wilmoth and Horiuchi (1999) discuss decreasing variances in ages at death in the context of exploring rectangularization of the human survival curve over the past several centuries. They describe large historical declines in variance concomitant with increasing life expectancy, which is fairly consistent with rectangularization. But they also find that the past several decades have seen decelerating variance paired with continuing increases in mean ages at death, suggesting that rectangularization has subsided over the past several decades.

Two recent papers discuss the importance of individual-level variance in understanding aggregate economic and demographic phenomena. Li and Tuljapurkar (2003) show that incorporating realistic changes in the variance of the age at death into a macroeconomic model with overlapping, finite-lived generations can significantly alter the model's predictions regarding economic effects of population aging. Edwards and Tuljapurkar (2003) compare postwar patterns of convergence among advanced economies in income per capita and in life expectancy, uncovering a puzzle: convergence in income is not closely related to convergence in longevity. Among the possible explanations for the puzzle is that differential individual-level heterogeneity

in health and mortality, such as reflected in variance in the age at death, may play a significant role. There are large differences across countries in variability in the age at death given the mean.

First appearances suggest that convergence in variances of the age at death is indeed occurring simultaneously and in a similar fashion as mean ages at death. Upon closer inspection, it turns out that much of the observed convergence in variance is due to convergence in infant and child mortality alone rather than to convergence along the entire distribution of the age at death. Patterns of joint convergence in unconditional averages and variances of the age at death among advanced countries appear likewise to be the result of declining early-age mortality rather than more general movement toward a single mortality distribution. Although mean ages at death display fairly strong convergence, key differences in variances in the age at death at older ages persist between advanced countries in the postwar period. These features are suggestive of underlying variation in the institutions associated with mortality decline, and thus they may help explain differential mortality decline among otherwise similar high-income countries.

This paper explores in greater detail the cross-country differences in both the mean and standard deviation of the age at death in a panel setting, assessing the relative importance of each in explaining mortality convergence among high-income countries. Section 2 explores postwar trends in the distributions of ages at death among industrialized economies by comparing conditional and unconditional means and variances of the age at death by country over time. In order to assess the relative importance of trends in means and variances for overall mortality convergence, decompositions of convergence based on the Kullback-Leibler Distance is presented in Section 3. Section 4 summarizes and suggests directions for future inquiry.

2 Postwar distributions of the age at death within low-mortality countries

Figure 1 shows four overlaid distributions of the age at death for both sexes combined. The data are lifetable deaths by single years of age up to 110 and over provided by the Human Mortality Database (2003), hereafter referred to as the HMD. Solid and dotted blue lines represent ages at death for Sweden in 1960 and 1996, while solid and dotted red lines show the same for the U.S.

Several key patterns are evident from the figure. First, the distributions are bimodal, with a spike in mortality at the youngest ages that represents infant and childhood mortality, and with a hump-shaped component at older ages that looks like a normal distribution with a thicker left tail. Second, the predominant pattern of change over time in both countries is the rightward movement of the distribution. Over this 36-year period, each distribution shifts rightward by between 5 and 10 years; this motion is the near-linear increase of the unconditional mean age at death that has averaged roughly 7 years over the time period, as reported by White (2002). Third, Figure 1 shows that in each of the two time periods, the peak of the right-hand mode of the Swedish distribution has exceeded the analogous U.S. peak. This graphical pattern indicates that Swedish mortality has been more tightly centered than U.S. mortality; variance in the age at death has been lower in Sweden than in the U.S. Fourth, the peaks of both distributions have risen over time. That is, variance in the age at death has decreased. in both countries.

Similar patterns are evident among the 12 other high-income countries in the HMD. Figure 2 depicts changes in life expectancy at birth since 1960. Although the general trend is one of linear increases, there are noteworthy differentials in the rates of longevity increase by country. Canada has steadily improved its relative position over time, although it has neither surpassed Sweden nor improved as fast as Japan. Denmark began the period with the fourth highest life expectancy but ended it dead last. Japan led in terms of growth rates during the entire period and has led in levels since the 1970s. Sweden has remained near the top during the entire interval, while the U.S. has ranked consistently around the bottom.

Figure 3 shows unconditional standard deviations in the age at death for the same set of countries during the same period. Appendix A describes how means and standard deviations are constructed using period lifetable elements. Across all countries, there have been large declines in standard deviations of the age at death measured over all ages. But again, some countries stand out. Canada began the period similar to the U.S. but rapidly diverged and is now more like other European countries and Japan. Denmark experienced fairly typical declines in standard deviations. Japan decreased its variance rapidly in the years before 1970 and then converged to Swedish levels. Sweden was the leader in 1960 and more or less retained its standing throughout the period. The U.S. has experienced the most variance in the age at death among this group of countries for the past quarter century.

As shown at the extreme left-hand side of Figure 1, decreases in infant mortality since 1960 have been rapid. They represent a key factor conceivably driving both the increases in period life expectancy and the decreases in variance in the age at death seen in Figures 2 and 3. Figure 4 shows period life expectancy at age 10, or the average age at death conditional on survival until age 10, over the same time period. Broad similarity between Figures 2 and 4 suggest that although declines in infant mortality are important, much of the increases in period life expectancy at birth within the panel are attributable to large declines in later-life mortality. Regardless of conditioning on any particular age, there have been remarkable increases in average remaining lifespan among these countries during the postwar period. These patterns are consistent with the findings of Lee and Carter (1992), who analyze the U.S. alone, and with those of Tuljapurkar, Li and Boe (2000), who examine the G-7 countries. Both papers uncover evidence of universal linear declines in log age-specific mortality rates at all ages among advanced countries.

A remarkably different pattern is shown in Figure 5, which plots standard deviations in the age at death conditional on reaching age 10 by country over time. Unlike the monotonic decreases in unconditional variances shown in Figure 3, variances conditional on early survival have not adhered to any identifiable trend during the postwar period. Country-specific trends are quite illuminating when compared with the relative performances in life expectancy shown in Figure 2. Canada experienced high conditional variance not much lower than that of the U.S. until roughly 1980, when variance began to decline dramatically. Denmark began the period among the leaders in variance but rapidly lost ground and remained fairly average. Japan decreased its older-age variance in the age at death quite rapidly up until 1980 and then leveled off. Sweden encountered some small increases but has remained among the leaders during the entire period. And the U.S. has continuously experienced the highest standard deviation in the age at death after age 10, and it has shown no signs of significant change in the postwar period.

The stark contrast between Figures 3 and 5 demonstrates the dangers of blindly comparing unconditional variance in the age at death across space and time without considering age-specific patterns. It is interesting that the same cannot be said of the average age at death, however. Patterns of relative performance in period life expectancy, which have proved to be quite baffling (Edwards and Tuljapurkar, 2003), bear striking similarity to these divergent trends in conditional variance in the age at death.

3 Decomposing mortality convergence

The previous section showed how the variance of the age at death has behaved similarly, in the case of the unconditional variance, or quite differently, in the case of the variance conditional on early-age survival, than the average age at death among advanced countries since 1960. In light of these results, it is natural to examine the relative importance of convergence in means versus convergence in variances for explaining overall convergence in mortality.

It would be surprising to find a large role for convergence in variance, of course. Since the mean is of first-order significance, changes in the mean are also of first-order significance. The present inquiry is primarily concerned with producing a more complete description of mortality convergence, one that might assist researchers by identifying possible leads into why there is differential mortality decline among similar countries.

The **Kullback-Leibler measure of divergence**, hereafter KLD, is a commonly used metric for assessing the similarity of probability distributions. Its formula is displayed in Appendix B. The KLD has the standard properties of a metric; e.g., it is nonnegative. It reduces to the Euclidean distance between two vectors when they are close. Since the KLD relies on the natural logarithm, it can accommodate huge differences in distributions without becoming dominated by a small number of widely scattered observations, much like a geometric mean. If the KLD is used on distributions generated by a Markov process with equilibrium states, then the difference in the KLD between steps in the Markov chain is nonincreasing and will be monotonically decreasing if the process is ergodic.

Choice of the baseline distribution for comparison purposes is somewhat arbitrary. The KLD can be used to assess convergence to a single country's mortality distribution at any point in time, or to a single distribution that is fixed over time. Proceeding with the former answers the question, "How close are countries coming to one another over time," while the latter answers, "How close are countries coming to a single distribution?" The distinction is fairly subtle, but the first measure will tend to amplify cross-country differences while the second dilutes them somewhat by combining changes over time with changes over space.

Figure 6 depicts the KLD over time using Sweden's unconditional distribution of ages at death in 1996 as the baseline. These measures suggest much overall convergence in mortality schedules during the postwar period, with all countries experiencing monotonically decreasing KLD's over time. Denmark

and the U.S. stand out as the top two countries at the end of the period, indicating less ultimate convergence to the Swedish baseline. The picture does not change qualitatively if another leading country, such as Japan, is used, or if the age at death distribution is truncated below age 10 to remove the impact of child mortality.

One method of decomposing the KLD into components attributable to means and variances is to confine analysis to the pseudo-normal distribution of ages at death past childhood seen in Figure 1. If the distribution of the age at death is approximately normal, then the KLD between two distributions p_1 and p_2, where p_1 is the baseline, can be approximated by a simple, convenient function of means and standard deviations:

$$K(p_1, p_2) = \log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2}{2\sigma_2^2} - \frac{1}{2} + \frac{(\mu_1 - \mu_2)^2}{2\sigma_2^2}. \quad (1)$$

Appendix B provides a step-by-step derivation of (1) and discusses a simple way to interpret its components as representing the effects of differential means versus differential variances.

Figures 7–11 depict decompositions of the postwar KLD relative to Sweden in 1996 for Canada, Denmark, Japan, Sweden, and the U.S. The approximate KLD, which does not tend to differ substantially from the true KLD, is shown in blue, while its additive components attributable to differences in means and variances are shown in red and green respectively. Qualitatively, the picture does not change significantly if the choice of the baseline is Japan in 1996. The relative closeness of the blue and red lines in each figure implies two patterns. Across all countries, levels of past divergence have been primarily attributable to divergence in mean ages at death. But at the same time, most postwar convergence is also attributable to convergence in means.

The behavior of the green lines reflects the very different postwar trends in variance relative to trends in means. In Canada, diverging variance in the age at death prior to 1980 offset some of the convergence in means during that period, but variance contributed to overall convergence after 1980. By the end of the period, variance accounted for all of the divergence in Canada. In Denmark, convergence in means has been much less dramatic while convergence in variances largely has not taken place. Japanese mortality convergence is primarily attributable to convergence in means, with all of the gains from converging variance exhausted fairly early. Sweden's comparison to itself in the future is interesting because it highlights the horizontal movement of conditional variance in that country; virtually all of the

action within Sweden has been confined to progression in the average age at death. Finally, the U.S. presents probably the most interesting case, where large differences in variance have been consistently responsible over time for divergence while differences in means have actually declined.

4 Conclusion

Distributions of the age at death among advanced countries have displayed much convergence during the postwar period. This paper has shown that most of the convergence in mortality is associated with convergence in mean ages at death. There also is much convergence in unconditional variances in the age at death across industrialized economies. Robust declines in infant mortality are likely part of the driving force behind each. But although mean ages at death have increased conditional on any age, the same does not appear to be true of variances. There are large and persistent differences between countries in variances in the age at death conditional on early survival.

These large differences in variances and in the individual-level heterogeneity they represent are suggestive of underlying gradients in the institutions of mortality decline across countries. It is unlikely to be mere coincidence that key outliers in terms of the growth in life expectancy, such as Denmark, Japan, and the U.S., should also be outliers in terms of variance. Future efforts to identify the sources of divergent individual-level heterogeneity in health and mortality are warranted. This study has found several key turning points in variance trends, such as Japan in the early postwar period and Canada around 1980, that suggest avenues for future inquiry.

A Conditional means and variances of the age at death

The period average age at death conditional on survival until age $\tilde{x}$, $\mu_{\tilde{x}}$, can be computed from lifetable deaths ${}_nd_x$ using the following formula, where n is the width of the age groups and X is the highest age in the lifetable:

$$\mu_{\tilde{x}} = E[x^d \mid x = \tilde{x}] = \left[\sum_{x=\tilde{x}}^{X} {}_nd_x \cdot (x + n/2)\right] / \left[\sum_{x=\tilde{x}}^{X} {}_nd_x\right]. \tag{2}$$

If desired, the ${}_nd_x$'s can be normalized to sum to 1 over the range of ages considered, but (2) does not require it. As can be seen, μ_x is a weighted average of the ages at death, or rather, the midpoints of the ages at death, so as not to zero-out deaths in the 0-1 interval.

Variance in the age at death conditional on survival until $\tilde{x}$, $\sigma^2_{\tilde{x}}$, can also be calculated using lifetable ${}_nd_x$'s as the weights:

$$\sigma^2_{\tilde{x}} = \left[\sum_{x=\tilde{x}}^{X} {}_nd_x \cdot ([x + n/2] - \mu_{\tilde{x}})^2\right] / \left[\sum_{x=\tilde{x}}^{X} {}_nd_x\right] \times \frac{N}{N-1}, \tag{3}$$

where N is the number of age groups; $N = X/n$. Again, age is measured at the midpoint of the interval.

B The Kullback-Leibler divergence between two normal distributions

Given two distributions with densities $p_1(x)$ and $p_2(x)$, the Kullback-Leibler measure of the distance of p_2 from p_1 is given by

$$K(p_1, p_2) = \int_{-\infty}^{\infty} p_1(x) \log\left(\frac{p_1(x)}{p_2(x)}\right) dx. \tag{4}$$

Suppose the p_i are normally distributed with mean μ_i and variance σ_i^2. Then

$$p_i(x) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right], \tag{5}$$

and the Kullback-Leibler measure can be rewritten

$$\begin{aligned}
K(p_1, p_2) &= \int_{-\infty}^{\infty} p_1(x) \log\left(\frac{\frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right]}{\frac{1}{\sigma_2\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right]}\right) dx \\
&= \int_{-\infty}^{\infty} p_1(x) \left(\log\frac{\sigma_2}{\sigma_1} + \frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right) dx \\
&= \int_{-\infty}^{\infty} p_1(x) \log\frac{\sigma_2}{\sigma_1}\, dx \\
&\quad + \int_{-\infty}^{\infty} p_1(x) \left[\frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right] dx \\
&= \log\frac{\sigma_2}{\sigma_1} + \int_{-\infty}^{\infty} p_1(x) \left[\frac{(x-\mu_2)^2}{2\sigma_2^2} - \frac{(x-\mu_1)^2}{2\sigma_1^2}\right] dx
\end{aligned} \tag{6}$$

To proceed further, use the substitution:

$$\begin{aligned}(x-\mu_2) &= (x-\mu_1)+(\mu_1-\mu_2)\\ (x-\mu_2)^2 &= (x-\mu_1)^2+(\mu_1-\mu_2)^2+2(x-\mu_1)(\mu_1-\mu_2) \qquad (7)\end{aligned}$$

Substituting (7) into the second term on the right in (6) produces

$$\int_{-\infty}^{\infty} p_1(x)\left[\frac{(x-\mu_1)^2}{2\sigma_2^2}+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}+\frac{2(x-\mu_1)(\mu_1-\mu_2)}{2\sigma_2^2}-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right]dx$$

Rearranging and expanding terms yields

$$\begin{aligned}
&= \int_{-\infty}^{\infty} p_1(x)\left[\frac{\sigma_1^2}{\sigma_1^2}\frac{(x-\mu_1)^2}{2\sigma_2^2}-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right]dx\\
&\quad+\int_{-\infty}^{\infty} p_1(x)\left[\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}+\frac{2(x-\mu_1)(\mu_1-\mu_2)}{2\sigma_2^2}\right]dx\\
&= \int_{-\infty}^{\infty} p_1(x)\left[\frac{\sigma_1^2}{\sigma_2^2}-1\right]\frac{(x-\mu_1)^2}{2\sigma_1^2}\,dx\\
&\quad+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}+\frac{(\mu_1-\mu_2)}{\sigma_2^2}\int_{-\infty}^{\infty} p_1(x)(x-\mu_1)\,dx\\
&= \frac{1}{2\sigma_1^2}\left[\frac{\sigma_1^2}{\sigma_2^2}-1\right]\int_{-\infty}^{\infty} p_1(x)(x-\mu_1)^2\,dx+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}\\
&= \frac{1}{2\sigma_1^2}\left[\frac{\sigma_1^2}{\sigma_2^2}-1\right]\sigma_1^2+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}\\
&= \frac{\sigma_1^2}{2\sigma_2^2}-\frac{1}{2}+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}, \qquad (8)
\end{aligned}$$

where the facts that $\int(x-\mu_1)p_1(x)\,dx=0$ and $\int(x-\mu_1)^2p_1(x)\,dx=\sigma_1^2$ have been used. Substituting (8) for the second term on the right-hand side of (6) produces a simple formula for the Kullback-Leibler distance of distribution 2 from distribution 1 when both are normal:

$$K(p_1,p_2)=\log\frac{\sigma_2}{\sigma_1}+\frac{\sigma_1^2}{2\sigma_2^2}-\frac{1}{2}+\frac{(\mu_1-\mu_2)^2}{2\sigma_2^2}. \qquad (9)$$

When p_1 and p_2 are approximately normal, (9) is an approximation of the true Kullback-Leibler divergence.

Suppose the variances of p_1 and p_2 are the same. Then the only source of divergence between the two distributions is the difference between their two means. Setting $\sigma_1 = \sigma_2$ in (9) produces a convenient formulation of the partial effect of divergent means:

$$K_\mu(p_1, p_2) = \frac{(\mu_1 - \mu_2)^2}{2\sigma_2^2}. \tag{10}$$

If the distributions have the same mean but different variances, (9) becomes

$$K_\sigma(p_1, p_2) = \log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2}{2\sigma_2^2} - \frac{1}{2}, \tag{11}$$

which can similarly be interpreted as the partial effect on the Kullback measure of divergent variances.

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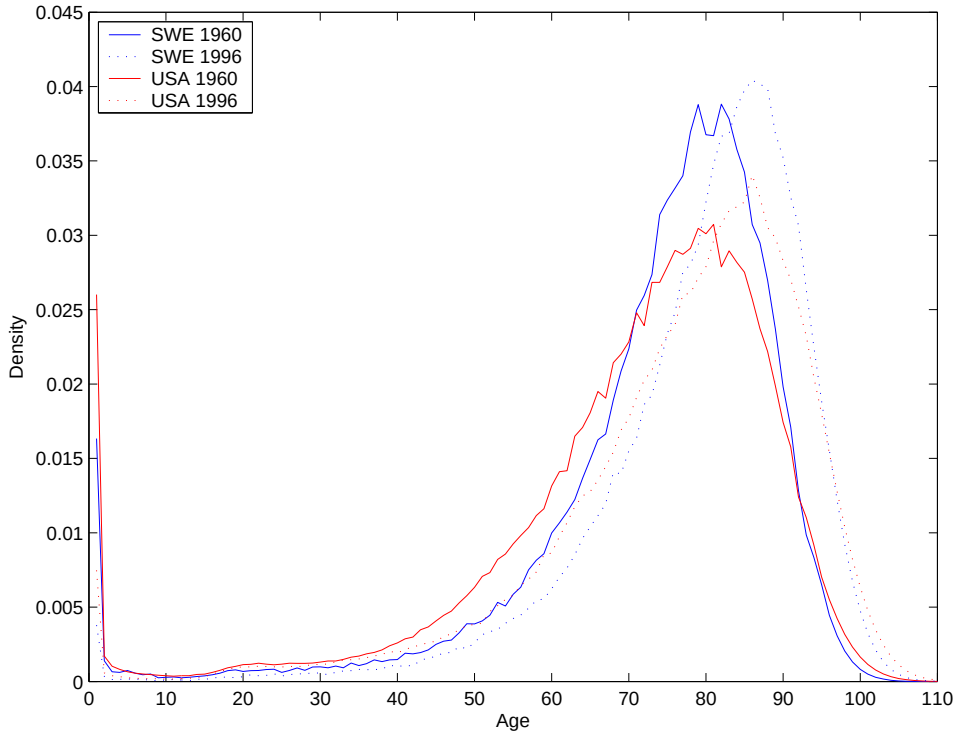
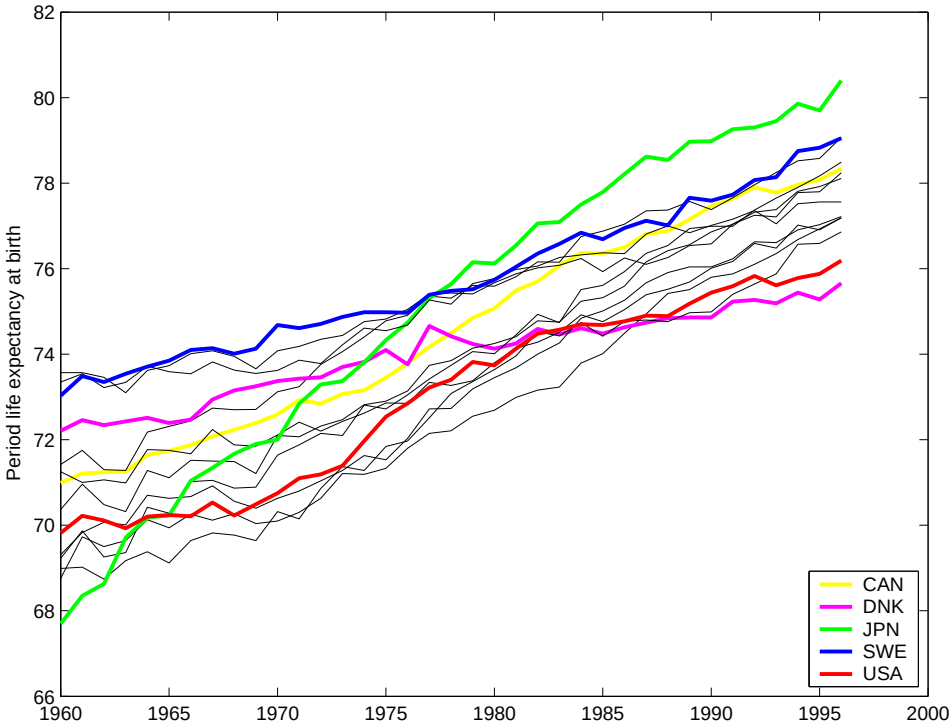
Figure 1: Age at death distributions for the U.S. and Sweden, 1960 and 1996



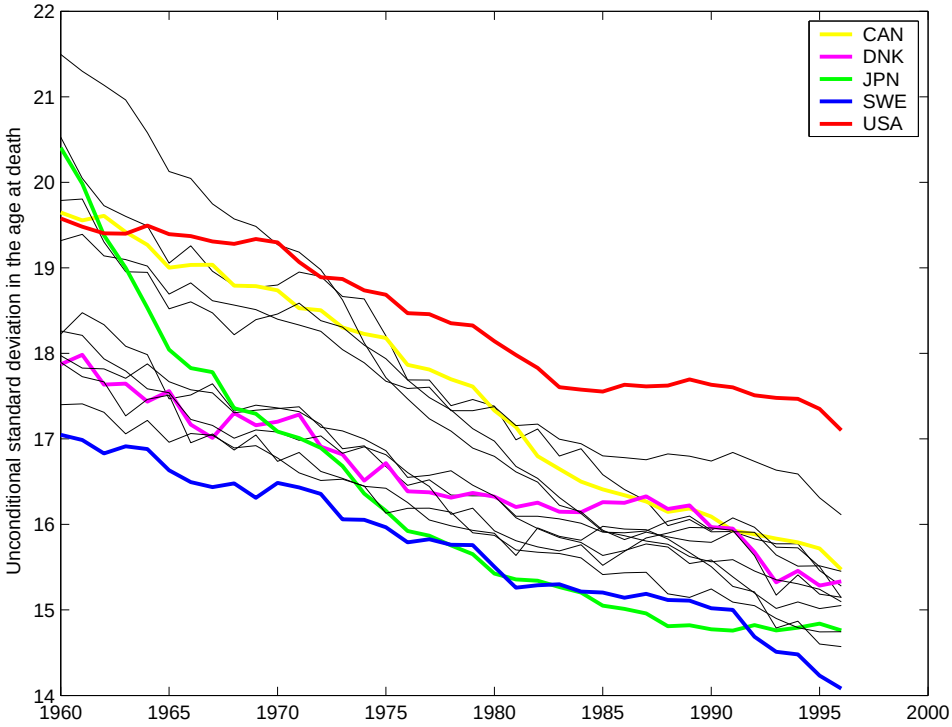
Notes: Data are lifetable deaths for both sexes combined by single year of age, rescaled to sum to unity. Lifetables are provided by the Human Mortality Database (2003).

Figure 2: Period life expectancy at birth in 14 countries since 1960



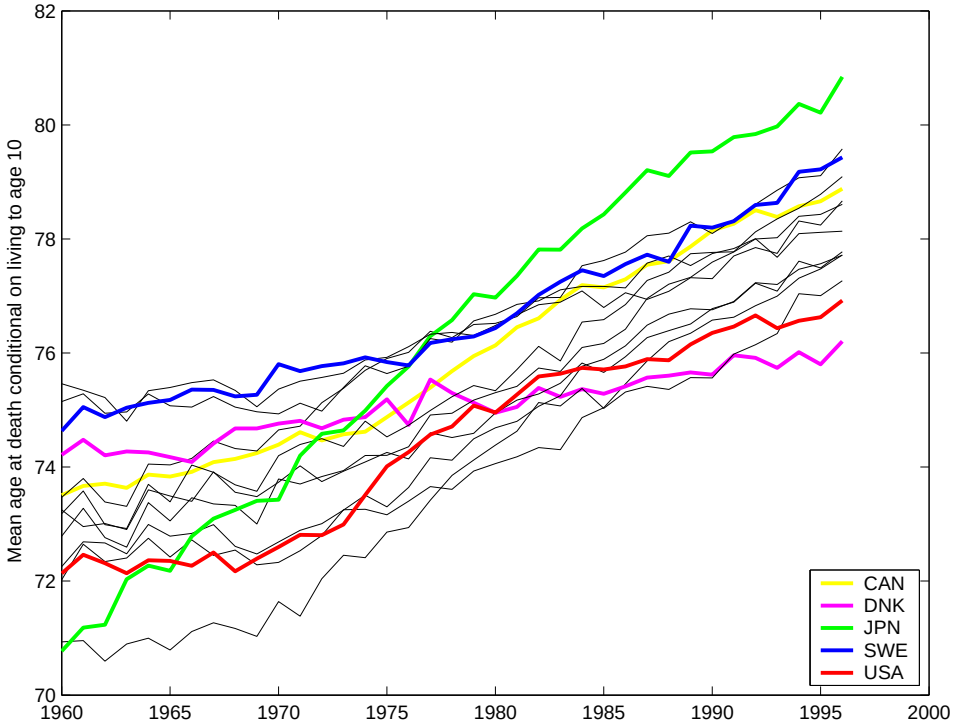
Notes: Data are period life expectancy at birth for both sexes combined. Lifetables are provided by the Human Mortality Database (2003). The 14 countries depicted are Austria, Canada, Switzerland, Denmark, Finland, France, England and Wales, Western Germany, Italy, Japan, the Netherlands, Norway, Sweden, and the U.S.

Figure 3: Unconditional standard deviations in the age at death in 14 countries since 1960



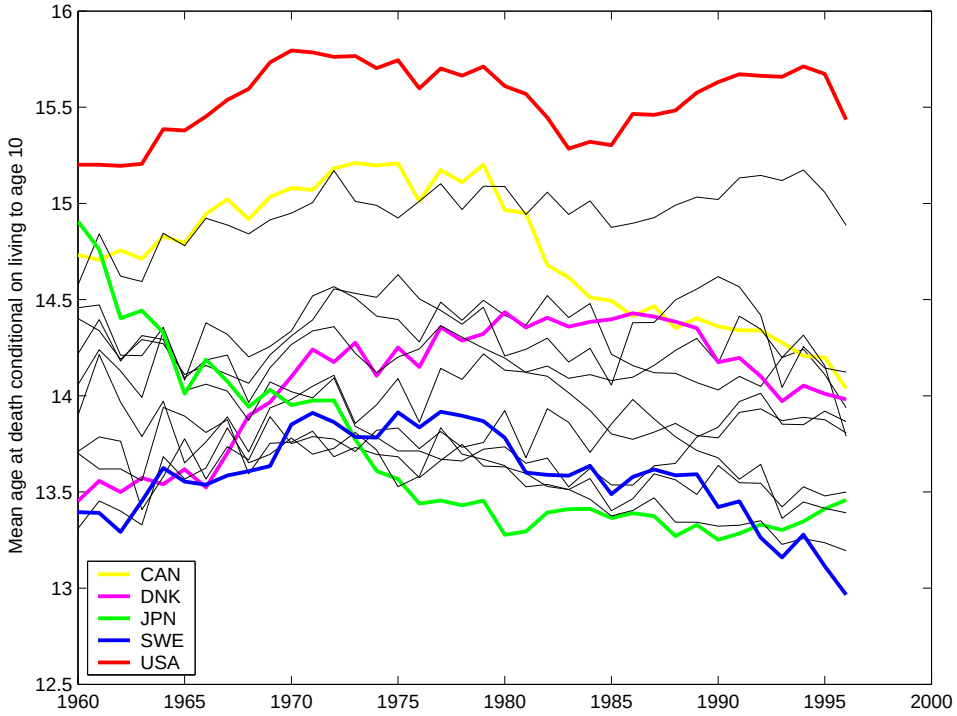
Notes: Data are the unconditional period standard deviation in the age at death using the entire lifetable distribution of ages at death. See Appendix A for details on how the data are constructed. Lifetables are provided by the Human Mortality Database (2003). The 14 countries depicted are Austria, Canada, Switzerland, Denmark, Finland, France, England and Wales, Western Germany, Italy, Japan, the Netherlands, Norway, Sweden, and the U.S.

Figure 4: Mean ages at death conditional on survival until age 10 in 14 countries since 1960



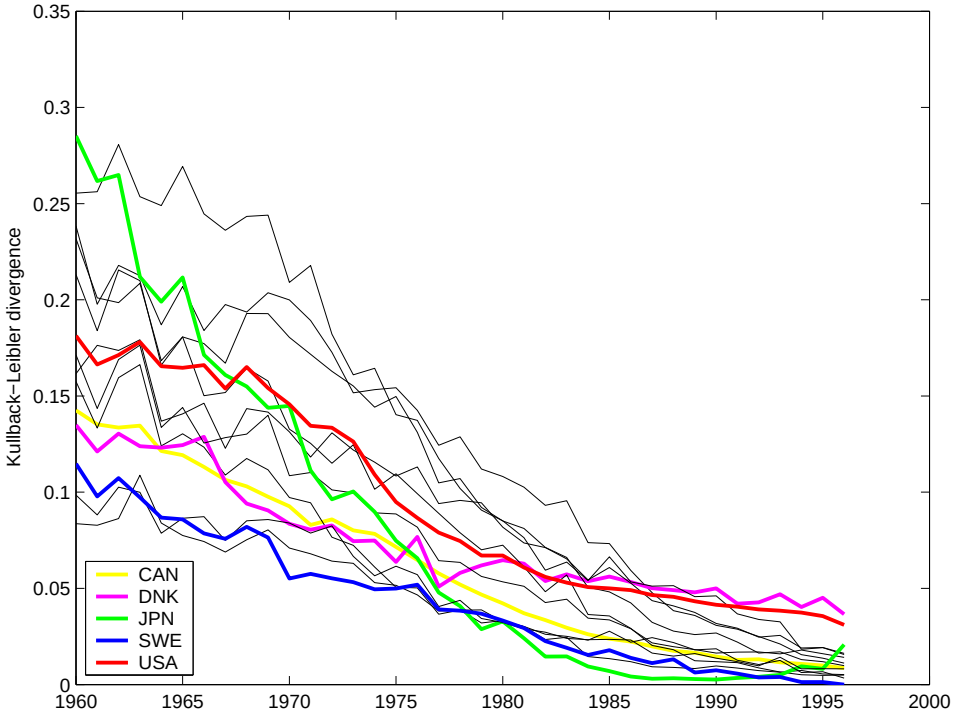
Notes: Data are averages ages at death constructed by truncating the distribution of ages at death for both sexes combined below age 10. See Appendix A for details. Lifetables are provided by the Human Mortality Database (2003). The 14 countries depicted are Austria, Canada, Switzerland, Denmark, Finland, France, England and Wales, Western Germany, Italy, Japan, the Netherlands, Norway, Sweden, and the U.S.

Figure 5: Standard deviations in the age at death conditional on survival until age 10 in 14 countries since 1960



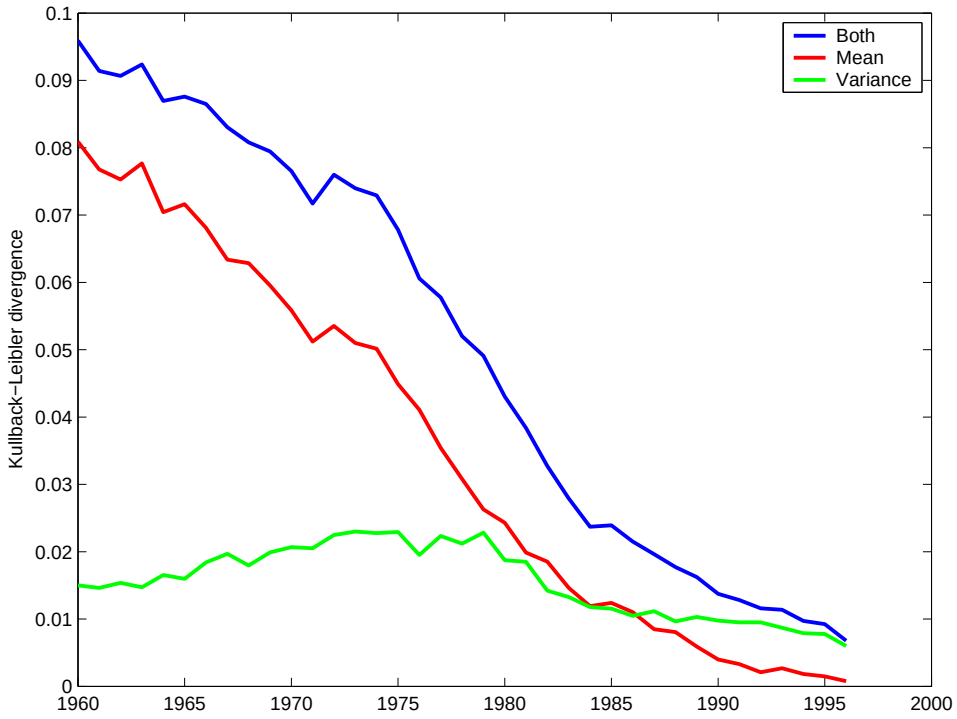
Notes: Data are the standard deviations in the age at death constructed by truncating the distribution of ages at death for both sexes combined below age 10. See Appendix A for details. Lifetables are provided by the Human Mortality Database (2003). The 14 countries depicted are Austria, Canada, Switzerland, Denmark, Finland, France, England and Wales, Western Germany, Italy, Japan, the Netherlands, Norway, Sweden, and the U.S.

Figure 6: Kullback-Leibler divergence between the unconditional distributions of the age at death in 14 countries since 1960 and Sweden's in 1996



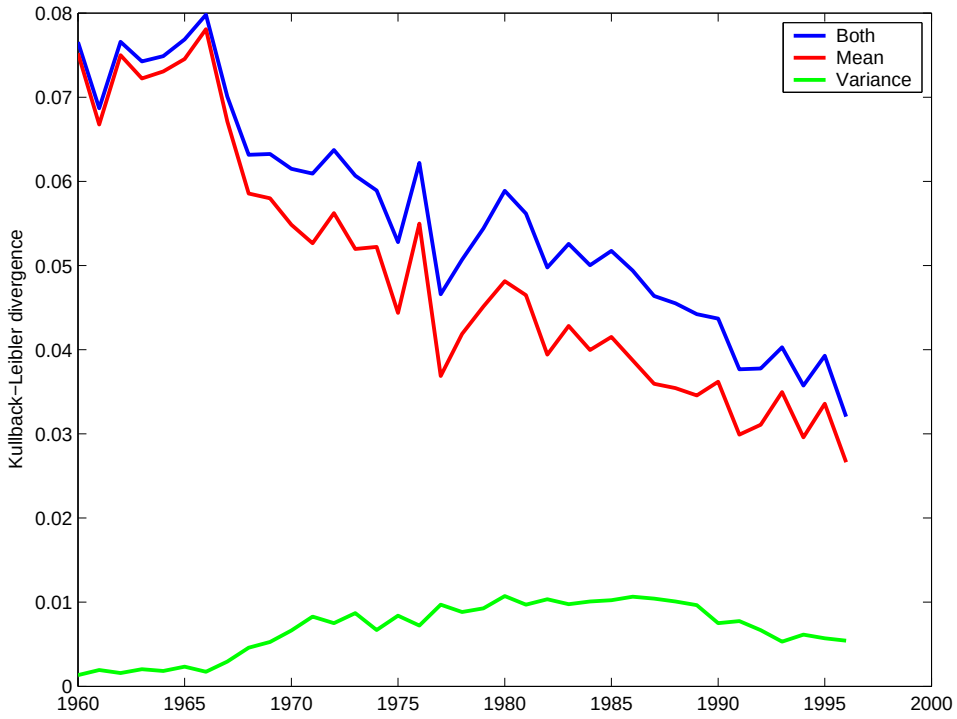
Notes: In each year, the measure is the Kullback-Leibler divergence (KLD) of each country's distribution of ages at death relative to Sweden's distribution in 1996. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).

Figure 7: Canada: Approximations of the Kullback-Leibler divergence between the distributions of the age at death since 1960 and Sweden's in 1996, conditional on survival to age 10



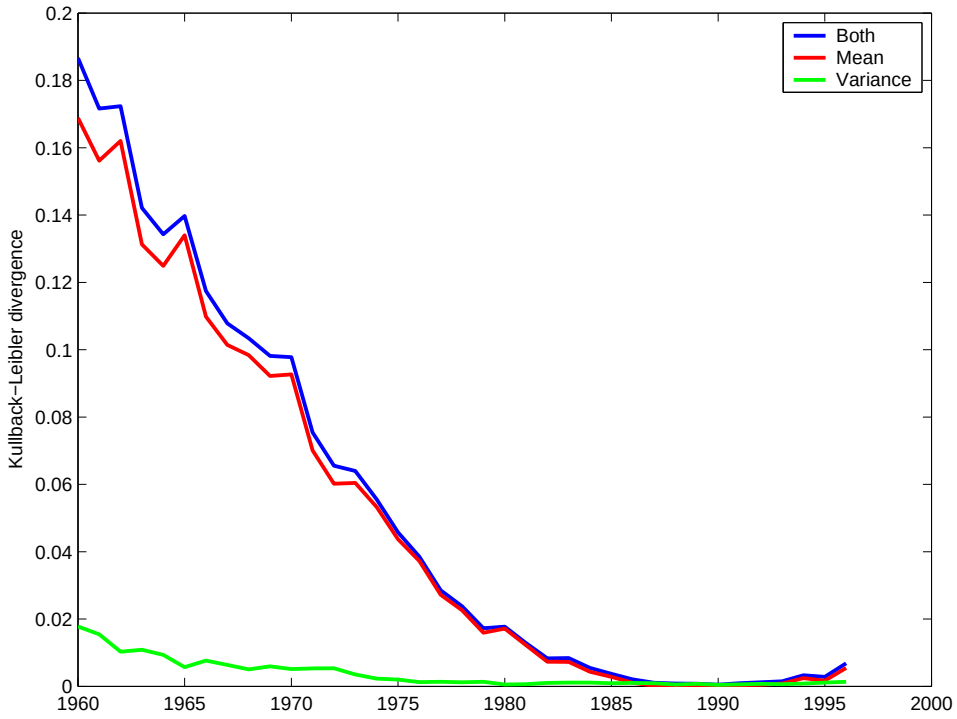
Notes: In each year, the approximate Kullback-Leibler divergence (KLD) of Canada's distribution of ages at death relative to Sweden's distribution in 1996 (blue line) is presented along with that part of the KLD attributable to differences in the mean (red line) and the part attributable to differences in the variance (green line). Distributions are truncated below age 10. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).

Figure 8: Denmark: Approximations of the Kullback-Leibler divergence between the distributions of the age at death since 1960 and Sweden's in 1996, conditional on survival to age 10



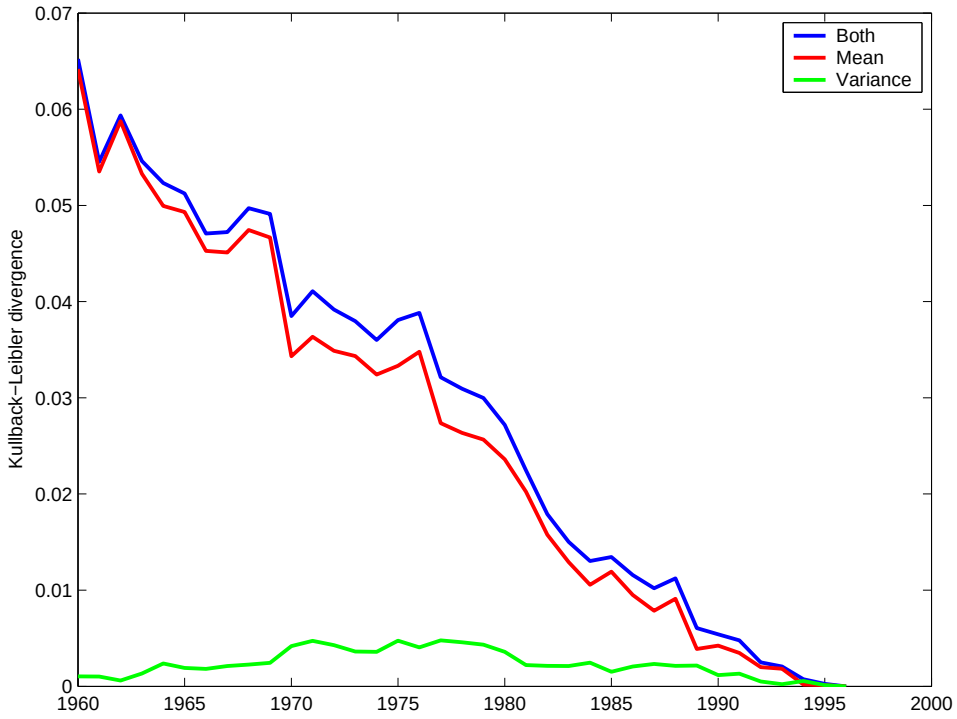
Notes: In each year, the approximate Kullback-Leibler divergence (KLD) of Denmark's distribution of ages at death relative to Sweden's distribution in 1996 (blue line) is presented along with that part of the KLD attributable to differences in the mean (red line) and the part attributable to differences in the variance (green line). Distributions are truncated below age 10. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).

Figure 9: Japan: Approximations of the Kullback-Leibler divergence between the distributions of the age at death since 1960 and Sweden's in 1996, conditional on survival to age 10



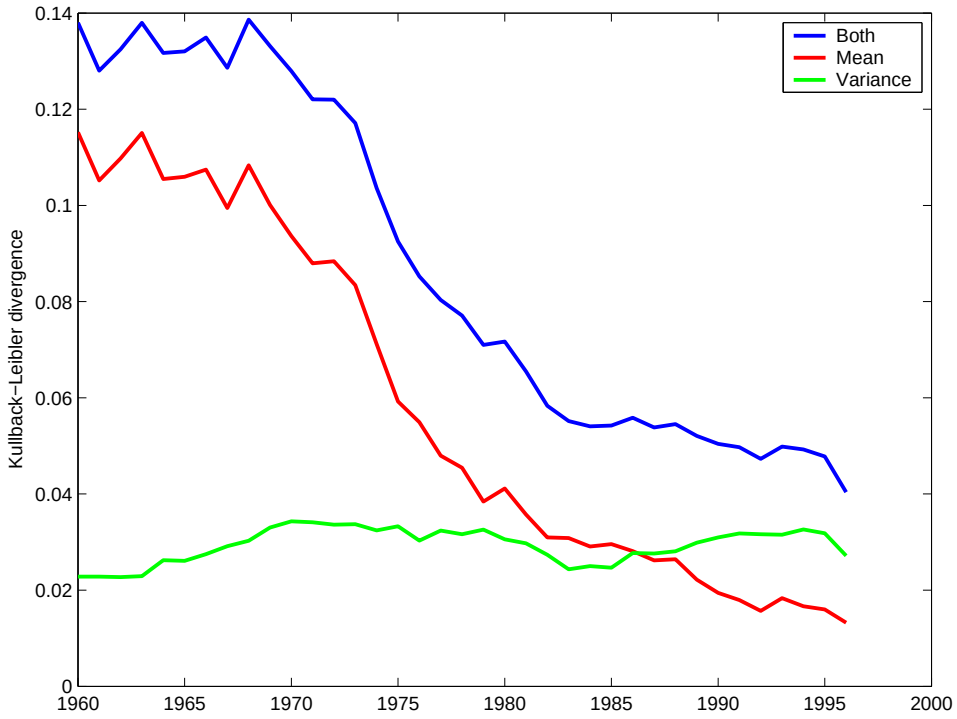
Notes: In each year, the approximate Kullback-Leibler divergence (KLD) of Japan's distribution of ages at death relative to Sweden's distribution in 1996 (blue line) is presented along with that part of the KLD attributable to differences in the mean (red line) and the part attributable to differences in the variance (green line). Distributions are truncated below age 10. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).

Figure 10: Sweden: Approximations of the Kullback-Leibler divergence between the distributions of the age at death since 1960 and Sweden's in 1996, conditional on survival to age 10



Notes: In each year, the approximate Kullback-Leibler divergence (KLD) of Sweden's distribution of ages at death relative to Sweden's distribution in 1996 (blue line) is presented along with that part of the KLD attributable to differences in the mean (red line) and the part attributable to differences in the variance (green line). Distributions are truncated below age 10. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).

Figure 11: U.S.: Approximations of the Kullback-Leibler divergence between the distributions of the age at death since 1960 and Sweden's in 1996, conditional on survival to age 10



Notes: In each year, the approximate Kullback-Leibler divergence (KLD) of the U.S. distribution of ages at death relative to Sweden's distribution in 1996 (blue line) is presented along with that part of the KLD attributable to differences in the mean (red line) and the part attributable to differences in the variance (green line). Distributions are truncated below age 10. See Appendix B for details on calculating the KLD. Lifetables are provided by the Human Mortality Database (2003).